The Smart Motion Cheat Sheet

$10.00

Smart Motion Systems are defined, for our purposes, as motion systems where speed, acceleration rate, and position (and sometimes torque) can be digitally programmed. Smart Motion Systems consist of three basic functional blocks: Brains, Muscle, and Load. The “Brains” (controls) selected will depend significantly upon application details, the features desired by the system designer or user, and personal preference. The “Load” and the motion mechanism used are dictated by the application requirements and the machine designer. But the “Muscle” (the motor & drive) is the essential element of a Smart Motion System where it is possible for a degree of science to take over. For an application with a given Load (and mechanism) with the appropriately selected Brains, as long as the <u>torque available</u> (at speed) from the selected motor-drive system exceeds the <u>torque required</u> to perform the desired motion, the application should be a success.

The Smart Motion Cheat Sheet was created to provide the system designer the information most commonly used to properly determine the Muscle (torque at speed) required by a given application and to give some guidelines for selecting the most appropriate motor-drive system to deliver that required torque at speed.

While it is desirable to have a basic knowledge of the different smart motion technologies currently available, it is not essential. What is essential is that the application requirements be well defined, that the torque at speed requirements be determined with a fair degree of accuracy, and that the Muscle (motor-drive) be selected based upon its ability to robustly deliver the required torque at speed. While it may be interesting and even useful, it is not essential to know what happens inside a given smart motor or drive in order to properly select and utilize it.

Having said that, a short discussion of the characteristics of the major commercially available Muscle for smart motion systems is appropriate. There are two commonly used classes of smart muscle: stepper systems and servo systems.

Stepper systems (motor & drive) are fundamentally open-loop systems which accept digital commands. They respond to digital step & direction inputs provided by an “indexer” or “motion controller” (Brain) which is basically a programmable pulse generator. This sequence of pulses is “translated” into motion of the motor by the drive (“translator”). The result is a very cost-effective all-digital Smart Motion System.

Stepper motors are brushless motors that include permanent magnet, variable reluctance, and hybrid types. Within these types there are many different variations of motor construction including 2-, 3-, 4-, and 5-phase windings with many different pole counts and mechanical step angles.

Overall, the function of the stepper drive is to sequentially regulate the current into the motor phase windings in order to produce the desired motion. The switching scheme used in a drive (full-, half-, mini-, micro-step) in combination with the mechanical construction of the motor determines the system resolution (steps/rev). While heat considerations ultimately limit the maximum torque from a given motor/drive system, the torque <u>at speed</u> is largely a function of the drive’s ability to overcome the inductance of the windings and push the maximum current into the phase windings as quickly as possible without over-heating. There are many different types of drives designed to accomplish this task (L/R, uni-polar, bi-polar, chopper, recirculating chopper, etc.) all of which have advantages. There is discussion of these in manufacturers’ literature.

For most stepper drives, being open loop by nature, the current sent to the motor is the same, independent of load variations. While many drives now provide a reduced current level when no motion is commanded, since motor current is always high, most get very hot, even when stopped. Another result of switching (commutating) current between windings without knowledge of the rotor velocity or position is to produce “resonance”. Resonance is the culmination of the complex open-loop dynamic interactions between motor, drive, load, and the commanded motion profile, and can reduce available torque significantly at some speeds.

An important characteristic of stepper systems (one frequently misunderstood) is that their commonly published torque vs. speed curves represents the torque at which system will stall <u>under ideal conditions</u>. Due to the resonance effects mentioned above, a stepper system will typically stall at 20-50% below this curve, depending upon speed. (See discussion on torque vs. speed curves on Page 5.)

Servo Systems: While stepper systems could be called a type of technology, "servo" is more properly a term, not a device or technology. A servo is by definition a "system" that makes corrections based upon feedback. It is also by definition "closed-loop". In the following discussion, we will be referring to servos as the many forms of electric motors and amplifiers (amp) used as closed-loop systems.

There are three basic loops in a Smart (positioning) electric servo system: the torque (current) loop; the velocity loop; and the position loop. The current loop is internal to the amp. Since there is a linear relationship between current and torque in (most) servo motors, the amp knows the torque being delivered from the motor based upon the current it is sending. Sensors on the motor and/or load provide velocity and/or position information to the amp and/or Brain. Sensors commonly used for both speed and position are encoders and resolvers. Earlier, tachometers were used for velocity, but advances in digital electronics allow deriving the velocity data from encoders and resolvers. Also, electronically commutated (brushless) motors require a commutation loop (feedback of rotor position in order to properly commutate).

Ultimately, the result of the "motion commands" coming from the Brain is to change the torque (current) sent to the motor in response to a deviation from the desired value of the measured speed and/or position. How much current (torque) should the amp send? It depends upon the error(s) between the desired speed and/or position, and upon the gains (amount of correction relative to amount of error) that are set (either by analog pots or digital settings) in the feedback loops. The higher the gain setting, the larger the change in the loop output for a given error.

To digress into an automotive analogy: Your car is a servo-system. It has a motor (engine), amplifier (carburetor), and Brain (cruise control & you or trip computer). It also has a torque loop (within the carburetor: engine output proportional to gas flow), velocity loop (speedometer and you, or cruise control), and position loop (odometer and you, or trip computer). Like an electric servo, if the speed or position differs from the desired, a change in torque is made. If you are a "high gain" driver (or if your carburetor and cruise control gains are high), your system can be high response. However, as with an electric servo, when the gains are too high for the load and motion profile, an "unstable" condition can result (wreck). Action: de-tune. If your system is sluggish for the load and the desired motion, increase the gains, or get a higher performance system.

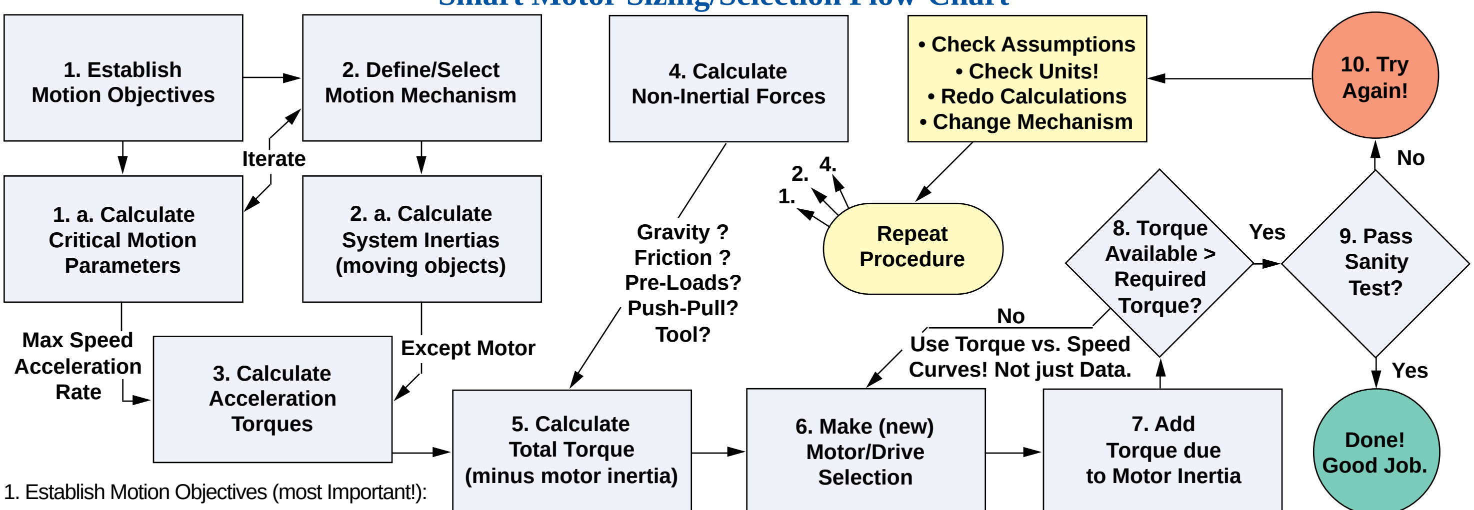
Most commercially available servos still use analog interfaces (not to be confused with analog hardware) to receive either velocity or torque commands from a "Brain". However, servos are increasingly becoming available with digital interfaces (not to be confused with digital hardware) which either emulate a stepper motor interface (and from the Brain viewpoint, can be controlled "open-loop" like stepper motors), or which receive torque, velocity, or position commands directly in a digital form.

Similar to steppers, there are a variety of implementations of electric servos, each of which have advantages. The more common distinguishing (or marketing) terms used for the various types of servos include: DC brush-type; AC brushless; DC brushless; Vector . . .; ECM (electronically commutated motors. . . . i.e. brushless), switched reluctance, synchronous servo, induction servo, etc. Some terms refer to motor construction; some to amplifier characteristics; some to both.

For more information on the differences between servo and stepper technologies, consult the manufacturers' literature, AIME, NEMA PMC Group, or attend a balanced generic class on Smart Motion.

Again, while the details of a given technology may be interesting and even helpful to know, as a system designer, your selection should not be based upon the technologies employed, but on their result: i.e., the torque at speed they robustly produce and their value (performance vs. cost) relative to your application requirements. When you take this approach, generally the most appropriate technology will select itself.

Smart Motor Sizing/Selection Flow-Chart



1. Establish Motion Objectives (most Important!):
   - Overall distance vs. time requirements?
   - Velocity vs. Time for entire cycle?
   - Worst-case move? (L distance in t time)
   - Any imposed max. speed constraints?
   - Required move resolution?
   - Required positioning repeatability?
   - Required positioning accuracy?

1. a. Calculate Critical Move Parameters:
   - Max. move speed ω_{max}?
   - Max. accel rate α?

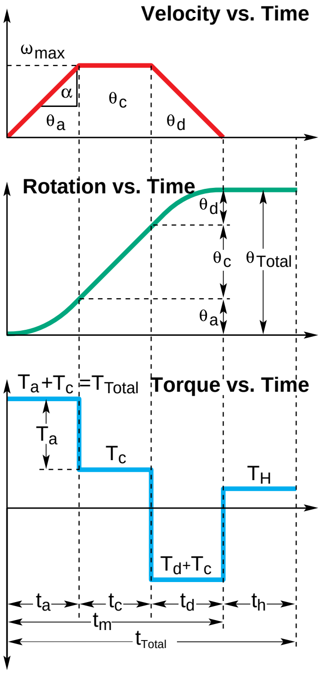
2. Define/Select Motion Mechanism:
   - Direct Drive? Screw? Tangential Drive?
   - Reducer? Type?

2. a. Calculate inertia of all moving components
   - Mechanism components; Reducer; Coupling
   - Reflect inertia's to motor

3. Calculate Acceleration Torque at motor shaft due to reflected inertia (load & mechanism only)
4. Calculate all non-inertial forces, torques
   - Forces, torques due to gravity?
   - Torques due to other external forces?
   - Friction? Pre-loads?
5. Calculate Total Torque reflected to motor
   - Acceleration/Inertial ($T=J_L\alpha$) torques
   - Plus all other Torques
   - Peak torque for worst case move
   - Also rms torque for entire move cycle
6. Make (initial) motor/drive selection
   - Torque available must exceed peak and rms
   - Remember, motor inertia hasn't been added
7. Calculate Torque added by motor inertia
   - Larger the accel rate = > higher significance

8. Torque Available exceeds Torque Required?
   - At all speeds?
   - Peak torque during accel?
   - RMS (continuous) over entire cycle?
   - Use Torque vs. Speed Curves, not just Data!
   - If No, return to 6, and select new motor
9. If Yes, does selection pass the "Sanity Test"?
   - "Sanity Test" = Does this make sense?
   - Forget the "numbers" . . . Use your common sense, intuition & judgment!
   - If Yes, you're done! Good Job! Implement!
10. If No, Try Again . . . Repeat the procedure
   - Double-check your assumptions
   - Redo your calculations
   - Triple-check your units!!
   - Try changing your mechanism

Key Motion Relationships



For Trapezoidal Moves

$$\theta_{Total} = \theta_a + \theta_c + \theta_d = \omega_{max} \times \left(\frac{t_a}{2} + t_c + \frac{t_d}{2}\right)$$

$$\omega_{max} = \frac{\theta_{Total}}{\left(\frac{t_a}{2} + t_c + \frac{t_d}{2}\right)}$$

For Triangular Moves **(if $t_c = 0$)**

$$\theta_{Total} = \theta_a + \theta_d = \omega_{max} \times \left(\frac{t_a}{2} + \frac{t_d}{2}\right)$$

$$\omega_{max} = \frac{\theta_{Total}}{\left(\frac{t_a}{2} + \frac{t_d}{2}\right)} \quad ; \text{ if } t_a = t_d,\ \omega_{max} = \frac{\theta_{Total}}{t_a}$$

Acceleration

$$\alpha = \frac{(\omega_{max} - \omega_o)}{t_a} \times 2\pi$$

NOTE: These formulas are easily derived knowing the area under the velocity vs. time curve is distance and its slope is acceleration. If you can calculate the area of rectangles, triangles, and the slope of a line (rise over run), you can remember and/or easily derive these formulas!!

Uniformly Accelerated Rotary Motion

Unknown	Known	Equation
θ (radians)	ω_o, t, α	$\theta = \omega_o t + \alpha t^2/2$
	ω_{max}, ω_o, t	$\theta = (\omega_{max} + \omega_o)t/2$
	ω_{max}, ω_o, α	$\theta = (\omega_{max}^2 - \omega_o^2)/(2\alpha)$
	ω_{max}, t, α	$\theta = \omega_{max} t - \alpha t^2/2$
ω_{max} (rad-sec^{-1})	ω_o, t, α	$\omega_{max} = \omega_o + \alpha t$
	θ, ω_o, t	$\omega_{max} = 2\theta/t - \omega_o$
	θ, ω_o, α	$\omega_{max} = \sqrt{\omega_o^2 + (2\alpha\theta)}$
	θ, t, α	$\omega_{max} = \theta/t + \alpha t/2$
ω_o (rad-sec^{-1})	ω_{max}, t, α	$\omega_o = \omega_{max} - \alpha t$
	θ, ω_{max}, t	$\omega_o = 2\theta/t - \omega_{max}$
	θ, ω_{max}, α	$\omega_o = \sqrt{\omega_{max}^2 - (2\alpha\theta)}$
	θ, t, α	$\omega_{max} = \theta/t - \alpha t/2$
t (sec)	ω_{max}, ω_o, α	$t = (\omega_{max} - \omega_o)/\alpha$
	θ, ω_{max}, ω_o,	$t = 2\theta(\omega_{max} + \omega_o)$
α (rad-s^{-2})	θ, ω_{max}, ω_o	$\alpha = (\omega_{max}^2 - \omega_o^2)/(2\theta)$
	ω_{max}, ω_o, t	$\alpha = (\omega_{max} - \omega_o)/t$
	θ, ω_o, t	$\alpha = 2(\theta/t^2 - \omega_o/t)$
	θ, ω_{max}, t	$\alpha = 2(\omega_{max}/t - \theta/t^2)$

Symbols & Definitions

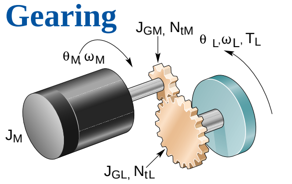
Symbol	Definition	Units SI	Units English
C_G	Circumference of Gear	m (or cm)	in (or ft)
$C_{P:1,2,3}$	Circumference of Pulleys, 1, 2, or 3	“	“
D	Diameter of cylinder or . . .	m (or cm)	in (or ft)
D_G	...(pitch dia.) of Gear	“	“
D_{PL}	...(pitch dia.) of Pulleys on Load	“	“
D_{PM}	...(pitch dia.) of Pulleys on Motor	“	“
$D_{P:1,2,3}$	...(pitch dia.) of Pulleys 1, 2, or 3	“	“
e	efficiency of mechanism or reducer	%	%
F	Forces due to...	N	lb
F_{tr}	...friction ($F_{fr} = \mu W_L \cos\gamma$)	“	“
F_g	...gravity ($F_g = W_L \sin\gamma$)	“	“
F_p	...Push or Pull forces	“	“
a or d	linear accel or decel rate	m-s^{-2}	in-s^{-2}
α	angular acceleration rate	rad-s^{-2}	rad-s^{-2}
g	gravity accel constant	9.80 m-s^{-2}	386 in-s^{-2}
J	mass moment of inertia for...	kg-m^2	lb-in^2
$J_{B\to M}$	...Belt reflected to Motor	or	or
J_C	...Coupling	g-cm^2	oz-in^2
J_G	...Gear	etc.	or
J_L	...Load	“	in-lb-s^2
$J_{L\to M}$	...Load reflected to Motor	“	or
J_M	...Motor	“	in-oz-s^2
J_{PL}	...Pulley on the Load	“	etc.
J_{PM}	...Pulley on the Motor	“	“
$J_{PL\to M}$	...Pulley on Load reflected to Motor	“	“
$J_{P:1,2,3}$	...Pulley or sprocket 1, 2, or 3	“	“
J_r	...reducer (or gearbox)	“	“
J_{Total}	...Total of all inertias	“	“
J_S	...lead Screw	“	“
N_r	Number ratio of reducer	none	none
N_t	Number of teeth on gear, pulley, etc.		
P_G	Pitch of Gear, sprocket or pulley	teeth/m	teeth/inch
P_S	Pitch of lead Screw	revs/m	revs/inch
t	time...	sec	sec
$t_{a,c,\text{ or }d}$	...for accel, constant speed or decel	“	“
t_m	...for move	“	‘
t_{Total}	...for Total Cycle	“	“
t_h	...for hold time (dwell time)	“	“

Symbol	Definition	Units SI	Units English
T	Torque...(for “required” Calculations)	Nm	in-lb
$T_{a,c,\text{ or }d}$	...during accel, constant, or decel	“	or
T_{CL}	...Constant at Load	“	in-oz
$T_{C\to M}$	...Constant reflected to Motor	“	“
T_H	...Holding (while motor stopped)	“	“
T_L	...at Load (not yet reflected to motor)	“	“
T_P	...due to Preload on screw nut, etc.	“	“
T_{RMS}	...RMS (“average”) over entire cycle	“	“
T_{Total}	...total from all forces	“	“
V_L	linear Velocity of Load	m-s^{-1}	in-s^{-1}
ω_O	initial angular/rotational velocity	rad-s^{-1}	rps or rpm
ω_M	angular/rotational velocity of Motor	“	“
ω_{max}	maximum angular/rotational velocity	“	“
W_L	Weight of Load	N (or kg)	lb
W_B	Weight of Belt (or chain or cable)	“	“
W_T	Weight of Table (or rack & moving parts	“	“
X_L	Distance X traveled by Load	m (or cm)	in (or ft)
θ	rotation...	radians	revs
$\theta_{a,c,\text{ or }d}$	...rotation during accel, decel, etc.	“	“
θ_L	...rotation of Load	“	“
θ_M	...rotation of Motor	“	“
θ_{Total}	Total rotation of motor during move	“	“
π	“PI” = 3.141592654	none	none
2π	rotational unit conversion (rads/rev)	rad/rev	rad/rev
μ	coefficient of friction	none	none
γ	load angle from horizontal	degrees	degrees
	The following Definitions apply to the Torque vs. Speed Curves		
	...typical torque terms used with servos..	Nm	in-lb
T_{PS}	Peak Torque at Stall (zero speed)	“	or
T_{PR}	Peak Torque at Rated Speed	“	in-oz
T_{CS}	Torque available continuously at Stall	“	“
T_{CR}	Continuous Torque Rating (@ rated speed)	“	“
	...typical torque terms used with Steppers...	“	“
T_H	Holding Torque (at zero speed)		
ω_R	Rated Speed (servos)	rad-s^{-1}	rps or rpm
ω_M	Maximum Speed (servos & steppers)		“
ω_1	Speed at Peak Torque (not commonly used)		
ω_H	“High” speed...real maximum (not common)	“	

Key Mechanism Related Equations

Motion Mechanism and Motion Equations	Inertia, Torque Equations	Other Factors To Consider

Gearing



Motion equations:

$$N_r = \frac{N_{tL}}{N_{tM}}$$

$$\theta_M = N_r \times \theta_L$$

$$\omega_M = N_r \times \omega_L$$

Inertia, torque equations:

$$J_{Total} = J_M + J_{GM} + J_{GL \to M} + J_{L \to M}$$

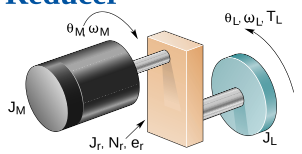
$$J_{GL \to M} = \left(\frac{1}{N_r}\right)^2 \times \frac{J_{GL}}{e} \qquad J_{L \to M} = \left(\frac{1}{N_r}\right)^2 \times \frac{J_L}{e}$$

$$T_{L \to M} = \frac{T_L}{N_r \times e}$$

Other factors to consider:

- Lubricant viscosity (oil or grease has major affect on drag torque!)
- Backlash
- Efficiency

Reducer



Motion equations:

$$N_r = \frac{\theta_M}{\theta_L} = \frac{\omega_M}{\omega_L}$$

$$\theta_M = N_r \times \theta_L$$

$$\omega_M = N_r \times \omega_L$$

Inertia, torque equations:

$$J_{Total} = J_M + J_r + J_{L \to M}$$

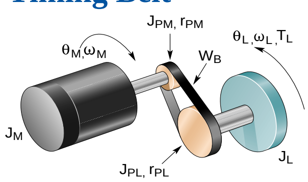
$$J_{L \to M} = \left(\frac{1}{N_r}\right)^2 \times \frac{J_L}{e} \qquad T_{L \to M} = \frac{T_L}{N_r \times e}$$

J_r = inertia of reducer reflected to input

Other factors to consider:

- Coupling inertia
- Gear and/or reflected reducer inertia

Timing Belt



Motion equations:

$$N_r = \frac{N_{TL}}{N_{tM}} = \frac{D_{PL}}{D_{PM}}$$

$$\theta_M = N_r \times \theta_L$$

$$\omega_M = N_r \times \omega_L$$

Inertia, torque equations:

$$J_{Total} = J_M + J_{PM} + J_{PL \to M} + J_{B \to M} + J_{L \to M}$$

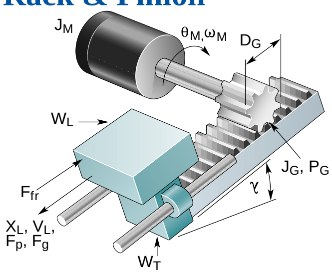
$$J_{PL \to M} = \left(\frac{1}{N_r}\right)^2 \times \frac{J_{PL}}{e} \qquad J_{B \to M} = \frac{W_B}{g \times e} \times \left(\frac{D_{PM}}{2}\right)^2$$

$$J_{L \to M} = \left(\frac{1}{N_r}\right)^2 \times \frac{J_L}{e} \qquad T_{L \to M} = \frac{T_L}{N_r \times e}$$

Other factors to consider:

- Pulley inertias
- Inertia is proportional to r^4 !
- Belt/chain inertia

Rack & Pinion



Motion equations:

$$C_G = \pi \times D_G = \frac{N_t}{P_G}$$

$$\theta_M = \frac{X_L}{C_G}$$

$$\omega_M = \frac{V_L}{C_G}$$

Inertia, torque equations:

$$J_{Total} = J_M + J_G + J_{L \to M}$$

$$J_{L \to M} = \frac{(W_L + W_T)}{g \times e} \times \left(\frac{D_G}{2}\right)^2$$

$$F_g = (W_L + W_T) \times \sin\gamma$$

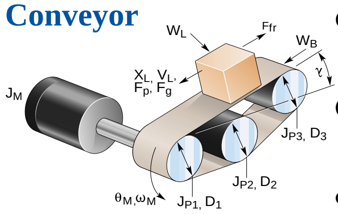
$$F_{fr} = \mu \times (W_L + W_T) \times \cos\gamma$$

$$T_{L \to M} = \left(\frac{F_P + F_g + F_{fr}}{e}\right) \times \left(\frac{D_G}{2}\right)$$

Other factors to consider:

- Backlash
- Pinion inertia
- Bearing friction
- Counter-balance vertical loads if possible
- Brake on vertical loads
- Linear bearing max speed limit

Conveyor



Motion equations:

$$C_{P1} = \pi \times D_{P1} = \frac{N_t}{P_G}$$

$$\theta_M = \frac{X_L}{C_{P1}}$$

$$\omega_M = \frac{V_L}{C_{P1}}$$

Inertia, torque equations:

$$J_{Total} = J_M + J_{P1} + \left(\frac{D_{P1}}{D_{P2}}\right)^2 \times \frac{J_{P2}}{e} + \left(\frac{D_{P1}}{D_{P3}}\right)^2 \times \frac{J_{P3}}{e} + J_{L \to M}$$

$$J_{L \to M} = \frac{(W_L + W_B)}{g \times e} \times \left(\frac{D_{P1}}{2}\right)^2$$

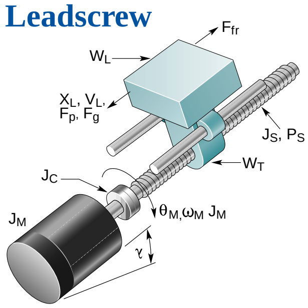
$$F_g = (W_L + W_B) \times \sin\gamma \qquad F_{fr} = \mu \times (W_L + W_B) \times \cos\gamma$$

$$T_{L \to M} = \left(\frac{F_P + F_g + F_{fr}}{e}\right) \times \left(\frac{D_{P1}}{2}\right)$$

Other factors to consider:

- Pulley inertias
- Belt/chain inertia
- Counter-balance vertical loads if possible
- Brake on vertical loads
- Linear bearing max speed limit

Leadscrew



Motion equations:

$$\theta_M = P_S \times X_L$$

$$\omega_M = P_S \times V_L$$

Inertia, torque equations:

$$J_{Total} = J_M + J_C + J_S + J_{L \to M}$$

$$J_{L \to M} = \frac{(W_L + W_T)}{g \times e} \times \left(\frac{1}{2\pi \times P_S}\right)^2$$

$$F_g = (W_L + W_T) \times \sin\gamma \qquad F_{fr} = \mu \times (W_L + W_T) \times \cos\gamma$$

$$T_{L \to M} = \left(\frac{F_P + F_g + F_{fr}}{2\pi \times P_S \times e}\right) + T_P$$

Other factors to consider:

- Screw inertia
- Coupling inertia
- Nut preload
- Bearing friction
- Leadscrew whip
- Max. ball speed
- Max. bearing speed

Typical Friction Coefficients ($F_{fr} = \mu W_L \cos\gamma$)

Materials	μ
Steel on Steel	~0.58
Stl. On Stl. (greased)	~0.15
Aluminum on Steel	~0.45
Copper on Steel	~0.30
Brass on Steel	~0.35
Plastic on Steel	~0.15-0.25

Mechanism	μ
Ball Bushings	<.001
Linear Bearings	<.001
Dove-Tail Slides	~0.2++
Gibb Ways	~0.5++

Material Densities

Material	gm/cm^3	lb/in^3
Aluminum	~2.66	~0.096
Brass	~8.30	~0.300
Bronze	~8.17	~0.295
Copper	~8.91	~0.322
Plastic	~1.11	~0.040
Steel	~7.75	~0.280
Hard Wood	~0.80	~0.029

Mechanism Efficiencies

Acme-screw w/brass nut	~0.35-0.65
Acme-screw w/plastic nut	~0.50-0.85
Ball-screw	~0.85-0.95
Preloaded Ball-Screw	~0.75-0.85
Spur or Bevel Gears	~0.90
Timing Belts	~0.96-0.98
Chain & Sprocket	~0.9[illegible]0.98
Worm Gears	~0.[illegible]0.85

Fundamental "Muscle" Selection Relationships

The fundamental relationship that must be met for a successful smart motion application is that the **Torque Available** (at all speeds) from the smart muscle (motor-drive system) must be **Greater Than** the **Torque Required** by the application.

$$T_{Available} > T_{Required} \text{ (at all speeds)}$$

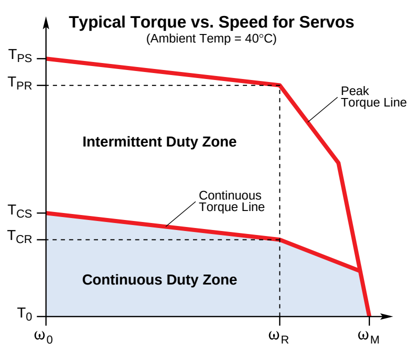
Thus, the procedure to follow is to first determine the total torque required (both Peak and Continuous or RMS), then compare it to the torque available from the motor-drive systems being considered. For available torque, use the motor-drive torque vs. speed performance curves whenever possible!!

1) $T_{Peak\,(Required)} = T_{TOTAL} = T_a + T_c$: Total Required Torque (Nm or in-lb) = Acceleration Torque (Nm or in-lb) + Constant Torques (Nm or in-lb)
   a. $T_a = J_{Total} * \alpha$: Acceleration Torque (Nm or in-lb) = Torque Inertia (kg-m^2 or in-lb-s^2) * Acceleration Rate (radians-sec^{-2})
      1. J_{Total} = motor inertia plus mechanism inertias reflected to motor (see formulas on Page 4)
      2. $\alpha = \omega_{max}/t_a * 2\pi$: Angular Acceleration (radians-sec^{-2}) = Max (or change in) Speed/accel time (rps/sec) * unit conversion (2π rad/rev)
   b. T_C = Torque due to all other non-inertial forces such as gravity, friction, preloads, tool, and other push-pull forces
      (VERY IMPORTANT: Use Consistent Units!! See unit conversions on Page 6)

2) $T_{RMS\,(Required)}$ = "Root Mean Squared": (~average) torque over entire cycle
   (refer to figures on page 3. Note: Watch your signs. . . As a vector quantity, $T_d = -T_a$)

$$T_{RMS} = \sqrt{\frac{(T_a = T_c)^2 \times t_a + T_c^2 \times t_c + (T_d + T_c)^2 \times t_d + T_h^2 \times t_h}{t_a + t_c + t_d + t_h}}$$

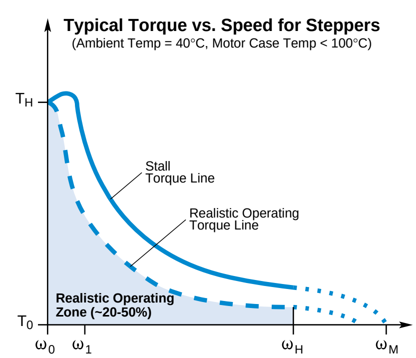
Interpretation of Servo & Stepper Torque vs. Speed Curves



Servos: The figure at left represents typical torque vs. speed curves for both brush and brushless electric servo systems. Servos typically have two zones: one in which continuous operation is possible; the second in which operation is possible only on an intermittent basis (from .05 to 30+ sec., depending on the manufacturer). Servos typically have a peak torque (either stall T_{PS} or rated T_{PR}) that is 2 to 3 times higher than the continuous torque (either stall T_{CS} or rated T_{CR}). Most makers list a "maximum speed" ω_M (usually 3000 to 6000 rpm) which would be the speed at full voltage and no load (T_0). Some makers list "rated" torques, which are the intersection of the Peak and Continuous Torque curves with a "rated speed" ω_R (commonly 3000+ rpm).

Since servos are closed-loop by definition, as long as the peak torque *required* is below the Peak Torque *(available)* Line and the *rms* torque required does not exceed the Continuous Torque Line, operation up to the Peak Torque Line is possible without fear of stalling or faulting.

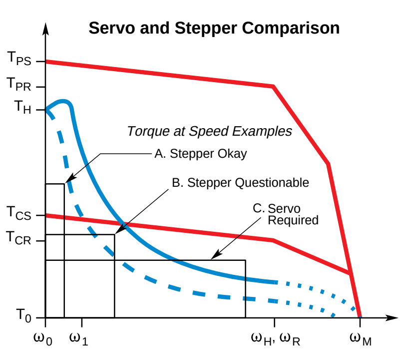
Key Considerations when comparing curves between various manufacturers with specific application include: Always try to use the torque vs. speed curves! If only tabular data is available, clearly understand what the data points represent. For example, is T_{max} at 0 speed or at max. speed? Etc. . . Is the curve for the motor <u>and drive</u> that you will be using? What ambient temperature is assumed (25° vs. 40° C makes a significant difference in real performance!)? Also, what voltage is assumed (available voltage affects the top speed)?



Steppers: Stepper motor-drive systems are used very successfully in many office and industrial automation applications. Properly applied they are typically the most cost-effective solution to a Smart Motion application. If their characteristics are mis-understood and they are mis-applied, costly applications failures frequently result.

The "Stall Torque Line" at left represents the typical ideal performance curve published by makers of stepper motors and drive systems. This curve must be interpreted very differently than servo curves. Due to the open-loop nature of stepper systems and the complex dynamic interactions between motor, drive, load, and motion profile, a stepper motor will frequently stall well before reaching this ideal stall torque line. And unless feedback is provided, the control system will not be able to respond. Also, even the ideal torque falls off rapidly above ω_1 (typically 100-600 rpm) to only 5-10% of holding torque T_H at ω_H (typically <3000 rpm).

Thus, when selecting stepper motor-drive systems, **unless an application is extremely well defined and the loads do not significantly vary**, it is recommended that the user use a reduced torque speed curve similar to the "Realistic Operating Line" shown at the left (which is somewhat arbitrarily defined as 50% of the Stall Torque Line). The resulting selections will be much more robust and your application will usually be much more successful.

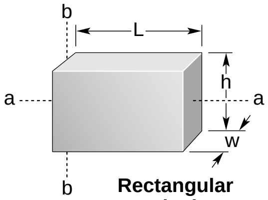


Steppers vs. Servos: If a stepper system will robustly perform an application, it will generally be lower cost than a "comparable" servo. The problem is defining a valid, consistent basis on which to compare them. The figure at left illustrates one basis on which to compare them. It is an over-lay of torque vs. speed curved. Also shown are the torque vs., speed requirements for 3 different application examples. Note that the holding torque T_H for the stepper system is over twice as much as the rated torque T_{CR} of the servo. Also note that the "maximum" speed for the stepper ω_M is greater than the rated speed of the servo ω_R.

Study of this figure will show that a selection based upon zero-speed torque alone (T_H vs. T_{CS} or T_{CR}, which is very common) will lead to erroneous conclusions. Application A shows that a stepper would be a better choice for low speed applications requiring fairly high continuous and/or peak torque. Application B illustrates that even at moderate speeds a stepper may not have the torque to do the same application that the servo shown can do even without utilizing the servo's intermittent torque. Application C is at higher speed and requires a servo, even though it requires less than a third of T_H and is at a speed less than ω_H of the stepper.

It can not be over-emphasized that comparisons of all systems should be done on the basis of realistic torque vs. speed information, not just holding or rated torque data!

Areas, Volumes, and Inertias for Simple Shapes



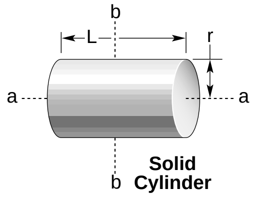
Rectangular Block

$A_{end} = h \times w$; $A_{side} = L \times h$; $V = L \times h \times w$

$$J_{a\text{-}a} = \frac{m}{12} \times \left(h^2 + w^2\right)$$

$$J_{b\text{-}b} = \frac{m}{12} \times \left(4L^2 + w^2\right) \quad \text{(if short)}$$

$$J_{b\text{-}b} = \frac{m}{3} \times \left(L^2\right) \quad \text{(if h \& w} \ll \text{L)}$$

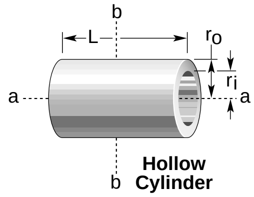


Solid Cylinder

$A_{end} = \pi \times r^2$; $V = A \times L$

$$J_{a\text{-}a} = \frac{mr^2}{2} = \frac{Wr^2}{2g} = \frac{\pi L \rho r^4}{2g}$$

$$J_{b\text{-}b} = \frac{m}{12} \times \left(3r^2 + L^2\right)$$



Hollow Cylinder

$A_{end} = \pi \times \left(r_0^2 - r_i^2\right)$; $V = A \times L$

$$J_{a\text{-}a} = \frac{m}{2} \times \left(r_0^2 + r_i^2\right)$$

$$= \frac{W}{2g} \times \left(r_0^2 + r_i^2\right) = \frac{\pi L \rho}{2g} \times \left(r_0^4 - r_i^4\right)$$

$$J_{b\text{-}b} = \frac{m}{12} \times \left(3r_0^2 + 3r_i^2 + L^2\right)$$

Common Units

Symbol	Definition	SI	Am/English
L	Length of solid	m or cm	in or ft
w	width of solid	m or cm	in or ft
h	height of solid	m or cm	in or ft
A	Area of shape	m^2 or cm^2	in^2 or ft^2
V	Volume of solid	m^3 or cm^3	in^3 or ft^3
W	Weight of solid	N	lbf
m	mass of solid	kg	lbm = lbf / g
$J_{a\text{-}a,\ b\text{-}b}$	Inertia about axis a-a, b-b	kg-m^2	in-lb=s^2 (& others)
r, r_0	outer radius	m or cm	in or ft
r_i	inner radius	m or cm	in or ft
g	accel or gravity, sea level	9.81 m-s^{-2}	386 in-s^{-2}
ρ	mass density of material	gm-cm^{-3}	lb-in^{-3} / g

General Formulae:

Mass: m = Weight / gravity (by definition, 1 N = 1 kg-m-s^{-2})
m (kg) = W (9.81 N) / g (9.81 m-s^{-2})
m (lbm) = m (lbf-s^2/386 in) = W (lbf) / g (386 in-s^{-2}) (sea level)

Weight: W = Volume * density (at sea level)
W (N) = V (cm^3) * ρ (gm-cm^{-3}) * (.001 kg/gm * 9.81 m-s^{-2})
W (lb) = V (in^3) * ρ (lb-in^{-3}/g) * (386 in-s^{-2})

Weight: W = max * gravity (at sea level)
W (N) = m (.102 kg) * g (9.81 m-s^{-2})
W (lb) = m (lb/386 in-s^{-2}) * g (386 in-s^{-2})

Common Engineering Unit Conversions

Parameter		System Intn'l (SI) Units		Common English/American Units	
Name	**Symbol**	**Unit**	**Name**	**Unit**	**Name**
Basic Units					
mass	m	kg	kilogram	lbm	pound mass
length (distance)	L	m	meter	ft (or in)	foot (or inch)
time	t	s	second	s	second
current	I	A	Ampere	A	Ampere
Derived Units					
Force (weight)	F (W)	N	Newton	lbf (or oz)	pound (or ounce)
Torque	T	Nm	Newton-meter	ft-lb (or in-lb)	foot-pound
Work (energy)	W (E)	J	Joule	ft-lb (or in-lb)	foot-pound
Power	P	W	Watt	hp (or W)	horsepower
Voltage, EMF	V	V	Volt	V	Volt
Resistance	R	Ω	ohms	Ω	ohms
Inertia	J	kg-m^2	kilogram-meter^2	in-lb-s^2 (+others)	inch-pound-second^2
plane angle	α, β, γ, etc.	rad	radian	deg or rad	degree or radian
rotation	θ	rev	revolution	rev	revolution
velocity (linear)	v	m-s^{-1}	meter per sec.	in-s^{-1}	inch per second
acceleration	a	m-s^{-2}	meter per sec.^{2}	in-s^{-2}	inch per second^2
velocity (angular)	ω	rad-s^{-1}	rad per second	rad-s^{-1}	rad per second
velocity (rotational)	ω	rpm	rev per minute	rpm	rev per minute
accel (angular)	α	rad-s^{-2}	rad per second^2	rad-s^{-2}	rad per second^2

Basic Definitions & Formulae

Definition/Formula		System Intn'l (SI) Units	English/American Units
Force (accel)	F = m * a	1 N = 1 kg * 1 m-s^{-2}	1 lbf = 1 lbf/(386 in-s^{-2}) * 386 in-s^{-2}
Torque (accel)	T = J * α	1 Nm = 1 kg-m^2 * 1 rad-s^{-2}	1 in-lb = 1 in-lb-s^2 * 1 rad-s^{-2}
Voltage (EMF)	V = I * R	1 V = 1 A * 1 Ω	1 V = 1 A * 1 Ω
Work (Energy)	E = F * L	1 J = 1 N * 1 m	1 in-lb = .113 Nm = .113 Ws = .113 J
Energy (elect.)	E = V * I * t	1 J = 1 V * 1 A*1 s	1 J = 1 V * 1 A * 1 s
Power	P = F * v	1 W = 1 N * 1 m-s^{-1}	1 hp = 550 ft-lb-s^{-1} = 745.7 W
or	P = T * ω	1 W = 1 Nm * 1 rad-s^{-1}	(note: radians are "unitless" values)
or	P = V * I	1 W = 1 V * 1 A	1 W = 1 V * 1 A
or	P = E * t^{-1}	1 W = 1 J * 1 s^{-1}	1 W = 1 J * 1 s^{-1}
or	P = I^2 * R	1 W = 1 A^2 * 1 Ω	1 W = 1 A^2 * 1 Ω
Motor Constants			
Torque Const.	K_t = T/I	K_t = Nm/A	K_t = in-lb/A
Voltage Const.	K_e = V/ω	K_e = V/(rad/s)	K_e = V/krpm
	(@ T = 0)	K_e = (V/(rad/s)) = K_t (Nm/A)	K_e (V/krpm) = 11.83 K_t (in-lb/A)
Servo Motor Formulae			
Current Draw	I = T * K_t^{-1}	1 A = 1 Nm * (Nm/A)$^{-1}$	1 A = 1 in-lb * (in-lb/A)$^{-1}$
Voltage Req'd	V = IR_a + K_e * ω	1 V = AΩ +V/(rad/s)*(rad/s)	1 V = AΩ +V/(krpm)*(krpm)

Common Unit Conversions

Length
1 in = .0254 m
1 in = 2.54 cm = 25.4 mm
1 in = 25,400 μm (microns)
1 μm = 39.37 * 10^{-6} in
1 ft = .3048 m; 1 m = 39.37 in
1 mile = 5280 ft
1 mile = 1.609 km

Mass, Weight, Force
1 lb = .453592 kg
1 lb = 4.44822 N
1 lb = 16 oz
1 kg = 9.81 N

Gravity Constant g (sea level)
g = 386 in-s^{-2} = 32.12 ft-s^{-2}
= 9.81 m-s^{-2}

Torque
1 in-lb = 16 in-oz = .113 Nm
1 ft-lb = 12 in-lb = 1.356 Nm
1 ft-lb = .138 kg-m
1 in-oz = .00706 Nm

Inertia
1 lb-in^2 = 2.93*10^{-4} kg-m^2
1 in-lb-s^2 = 0.113 kg-m^2
1 oz-in^2 = 1.83*10^{-5} kg-m^2
1 in-oz-s^2 = 7.06*10^{-3} kg-m^2
1 lb-ft^2 = 4.21*10^{-2} kg-m^2
1 ft-lb-s^2 = 1.355 kg-m^2
1 kg-cm^2 = 10^{-4} kg-m^2

Rotation
1 rev = 360 deg
1 rev = 2π radians
1 rev = 21,600 arc-min
1 rev = 1.296*10^6 arc-sec

Energy
1 in-lb = .113 Nm = .113 J
1 BTU = 1055 J
1 BTU = 252 calories

Power
1 hp ~ 746 W = 746 J-s^{-1}
1 hp = 550 ft-lb-s^{-1}
1 hp ~ 5250 ft-lb-rpm

SI Prefixes & Multiples

Tera	T	10^{12}
Giga	G	10^9
Mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deka	da	10^1
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	ρ	10^{-12}

To Convert Units Multiply by 1
if 1 lb = 16 oz,
then 1 = 16 oz/lb
or 1 = .0625 lb/oz

Example:
5 lb = ? oz.....
5 lb * (16 oz/lb) = 80 oz

Converting Inertia
Don't confuse mass inertia with weight inertia. Mass inertia is weight inertia divided by gravity constant "g"...

in-lb-s^2 (mass inertia) = lb-in^2/(386in/s^2)

Note: radians are "unitless" values!

Hint: convert to SI units and all will come out correctly!